NONSTATIONARY WAVES PROPAGATING ALONG A MAGNETIC FIELD IN A PLASMA

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The results of a numerical solution of the problem of the propagation of shock waves along a magnetic field in a cold rarefied plasma are presented. The parameters of the shock wave in the quasi-stationary phase for small Mach numbers $M \leqslant 2$ are presented. For values $M_* \approx 4$ the velocity profiles and the particle densities tend to become discontinuous.

Stationary solitary waves which propagate along the magnetic field in a cold plasma have been considered in [1-3].

NOTATION

c – velocity of light
m_e – electron mass
m_i – mass of an ion
β – ratio of the electron mass to the ion mass
t – time
e – charge of the electron
ω_{0i} – plasma frequency
H – magnetic field strength
$\omega_\sim$ – frequency of the magnetic field at the plasma-vacuum boundary
V_A – Alfvén velocity
V – volume in units of N_0^{-1}
N – particle density
ξ – Lagrange coordinate of the particles in units of c/ω_{0i}
τ – time in units of $c/(\omega_{0i}V_A)$
ν_{eff} – effective collision frequency
$\varkappa$ – collision frequency in units of ω_{eH}
u_p – mass velocity of the particles
u – mass velocity of the particles in units of V_A
x_0 – Euler coordinate of the particles in units of c/ω_{0i},
ω_{iH} – cyclotron frequency of the ions
ω_{eH} – cyclotron frequency of the electrons
ω – frequency of the magnetic field in units of ω_{iH}
ξ_{max} – coordinate of the plane of symmetry in units of c/ω_{0i}
$u_{x,y,z}$ – projections of the mass velocity of the particles on the x, y, and z axes in units of V_A
$h_{y,z}$ – projections of the magnetic field on the y and z axes in units of H_0
Δ – width of the wave front in units of c/ω_{0i}
Δ_{ux} – width of the particle velocity front in units of c/ω_{0i}
Δ_N – width of the particle density front in units of c/ω_{0i}
$h_\perp$ – transverse magnetic field in units of H_0

At the initial instant of time the cold quasi-neutral uniform plasma with density N_0 fills the half-space $x > 0$ (the x axis is in the direction of the unperturbed magnetic field H_0). Then, at the boundary of the plasma $x = 0$ the z component of the magnetic field starts to increase according to a certain law, as a result of which plane perturbations propagate along the x axis. The initial system of equations, written for convenience in dimensionless variables and Lagrange coordinates, has the following form:

$$\frac{\partial u_x}{\partial \tau} = -\frac{1}{2}\frac{\partial}{\partial \xi}(h_y^2 + h_z^2), \quad \frac{\partial u_{y,z}}{\partial \tau} = \frac{\partial h_{y,z}}{\partial \xi}$$

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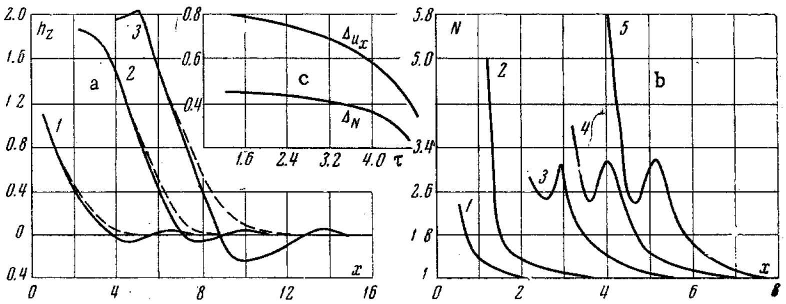

Fig. 1

$$
\begin{gathered}
u_x = \frac{\partial x_0}{\partial \tau}, \quad V = \frac{\partial x_0}{\partial \xi} \\
\frac{\partial}{\partial \tau}(Vh_y) = \frac{\partial u_y}{\partial \xi} + \frac{\partial^2 h_z}{\partial \xi^2} + \varkappa \frac{\partial^2 h_y}{\partial \xi^2} + \beta \frac{\partial^3 h_y}{\partial \tau \partial \xi^2} \\
\frac{\partial}{\partial \tau}(Vh_z) = \frac{\partial u_z}{\partial \xi} - \frac{\partial^2 h_y}{\partial \xi^2} + \varkappa \frac{\partial^2 h_z}{\partial \xi^2} + \beta \frac{\partial^3 h_z}{\partial \tau \partial \xi^2}
\end{gathered} \tag{1}
$$

$$
h = \frac{H}{H_0}, \quad V = \frac{N_0}{N}, \quad u = \frac{u_p}{V_A}, \quad x_0 = \frac{x\omega_{0i}}{c}
$$

$$
\tau = \frac{V_A \omega_{0i}}{c} t, \quad \beta = \frac{m_e}{m_i}, \quad \varkappa = \frac{\nu_{eff}}{\omega_{eH}}
$$

$$
\omega_{0i} = \left(\frac{4\pi N_0 e^2}{m_i}\right)^{1/2}, \quad \omega_{eH} = \frac{eH_0}{m_e c}, \quad V_A = \frac{H_0}{(4\pi N_0 m_i)^{1/2}}
$$

where ν_{eff} is the effective collision frequency, which is assumed to be constant.

This system of equations is obtained as a particular solution of system (1.4) in [4]. To solve the problem we assume the following initial and boundary conditions:

$$
\begin{gathered}
u_x(\xi, 0) = u_y(\xi, 0) = u_z(\xi, 0) = 0, \quad x_0(\xi, 0) = \xi \\
V(\xi, 0) = 1, \quad h_z(\xi, 0) = h_y(\xi, 0) = 0, \quad h_y(0, \tau) = 0 \\
h_z(0, \tau) = Af(\tau), \quad \frac{\partial h_z}{\partial \xi}(\xi_{\max}, \tau) = \frac{\partial h_y}{\partial \xi}(\xi_{\max}, \tau) = 0 \\
A = H_\sim / H_0 = \text{const}
\end{gathered} \tag{2}
$$

Here A is the amplitude of the magnetic field. The function $f(\tau)$ is taken in the form

$$
f(\tau) = 1 - \exp(-\omega\tau) \quad \text{or} \quad f(\tau) = \sin \omega\tau, \quad \omega = \omega_\sim / \omega_{iH}
$$

Problem (1), (2) was solved on a computer using a difference scheme of the second order of accuracy. Typical profiles of the magnetic field as a function of the Euler coordinate x for small Mach numbers $M \leqslant 2$ at successive instants of time are shown in Fig. 1a (the continuous lines are for h_z, and the broken lines are for $h_\perp = \sqrt{h_y^2 + h_z^2}$). Curves 1, 2, and 3 correspond to $\tau = 2.4$, 4.8, and 6.4. The calculations were carried out for $\varkappa = 0.2$, A = 2, and M = 1.45. For these values we calculated the particle density profiles at different instants of time; these are shown in Fig. 1b, where curves 1, 2, 3, 4, and 5 correspond to $\tau = 2.4$, 4.0, 4.8, 5.6, and 6.4.

In agreement with the law of the dispersion of waves [5] which propagate along the magnetic field in the region of frequencies $\omega \sim \omega_{eH}$, the profiles of the transverse components of the magnetic field have an oscillatory form. The spatial period of the oscillations is of the order of c/ω_{0i}. The phase shift between the z component and the y component of the magnetic field is 90°. For comparatively low Mach numbers, the shock wave which is formed is characterized by approximately constant front width Δ, since the nonlinear effects are compensated by dissipative and dispersion effects. Calculations with $\omega = 0.25$ and $\varkappa = 0.2$ show that an increase in the amplitude of the magnetic field leads to an increase in the velocity of the steady-state shock wave; thus, the values M = 1.4, 1.45, and 2.0, and $\Delta = 4.6$, 4.0, and 3.0, correspond to the values A = 1.5, 2.0, and 3.0.

A continuous increase in the density occurs while the wave is being formed. The bend which occurs on the density profile after a certain time corresponds to departure of the wave from the piston. A further increase in the magnetic field at the boundary leads to a sharp pileup of the plasma, the result of which is a discontinuity in the density in the region of the piston.

An increase in the amplitude of the magnetic field at the boundary leads to very nonstationary wave conditions; the slope of the particle density profile and the x component of the velocity increase considerably (Fig. 1c). This rearrangement of the wave structure indicates that an inversion stage is being approached. For example, for the case $A = 8$, and $\varkappa = 0.5$, the critical Mach number M_*, at which the above phenomena occur, is approximately 4.

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